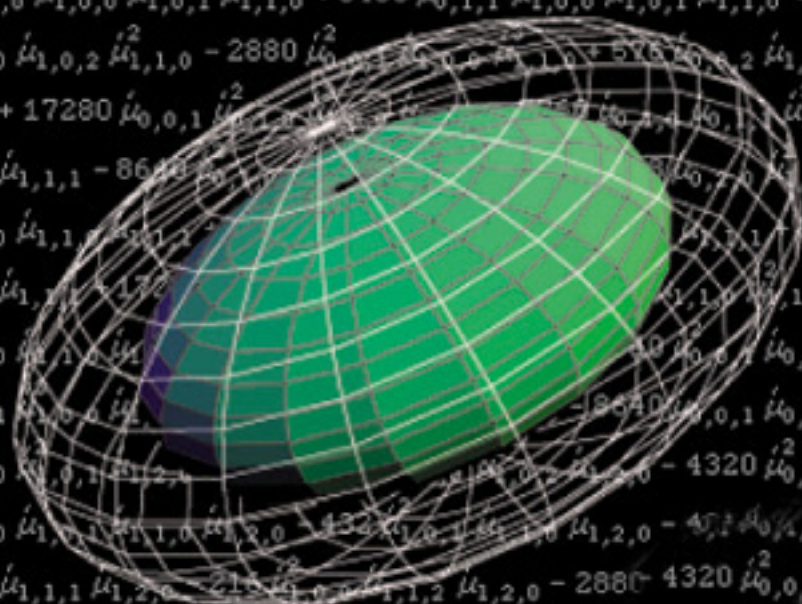


SPRINGER TEXTS IN STATISTICS

MATHEMATICAL STATISTICS

with
Mathematica[®]



COLIN ROSE
MURRAY D. SMITH

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