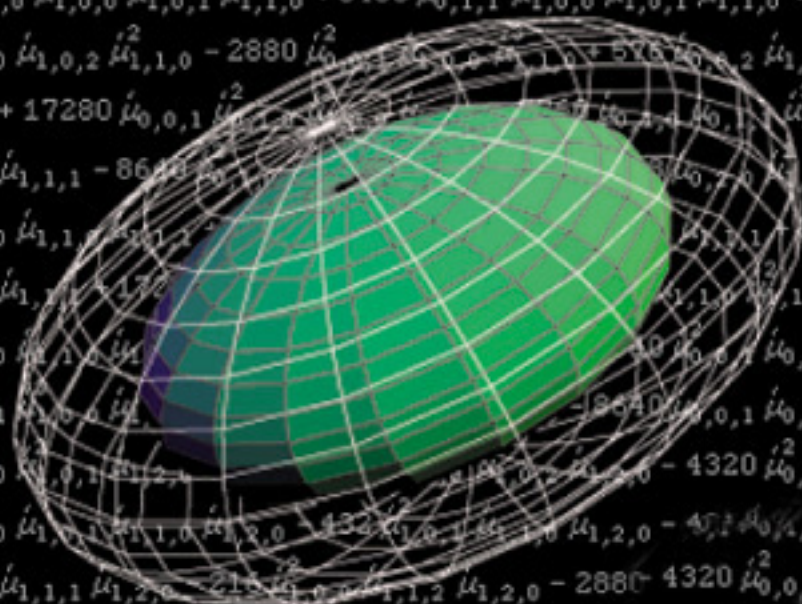


SPRINGER TEXTS IN STATISTICS

MATHEMATICAL STATISTICS

with
Mathematica[®]



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Preface

Imagine computer software that can find expectations of *arbitrary* random variables, calculate variances, invert characteristic functions, solve transformations of random variables, calculate probabilities, derive order statistics, find Fisher's Information and Cramér–Rao Lower Bounds, derive symbolic (exact) maximum likelihood estimators, perform automated moment conversions, and so on. Imagine that this software was wonderfully easy to use, and yet so powerful that it can find corrections to mainstream reference texts and solve new problems in seconds. Then, imagine a book that uses that software to bring mathematical statistics to life ...

Why *Mathematica*?

Why “Mathematical Statistics with *Mathematica*”? Why not Mathematical Statistics with Gauss, SPSS, Systat, SAS, JMP or S-Plus ... ? The answer is four-fold:

(i) *Symbolic engine*

Packages like Gauss, SPSS, *etc.* provide a numerical/graphical toolset. They can illustrate, they can simulate, and they can find approximate numerical solutions to numerical problems, but they cannot solve the algebraic/symbolic problems that are of primary interest in mathematical statistics. Like all the other packages, *Mathematica* also provides a numerical engine and superb graphics. But, over and above this, *Mathematica* has a powerful symbolic/algebraic engine that is ideally suited to solving problems in mathematical statistics.

(ii) *Notebook interface*

Mathematica enables one to incorporate text, pictures, equations, animations and computer input into a single interactive live document that is known as a ‘notebook’. Indeed, this entire book was written, typeset and published using *Mathematica*. Consequently, this book exists in two identical forms: (a) a printed book that has all the tactile advantages of printed copy, and (b) an electronic book on the **mathStatica** CD-ROM (included)—here, every input is live, every equation is at the reader's fingertips, every diagram can be generated on the fly, every example can be altered, and so on. Equations are hyperlinked, footnotes pop-up, cross-references are live, the index is hyperlinked, online HELP is available, and animations are a mouse-click away.

(iii) *Numerical accuracy*

Whereas most software packages provide only finite-precision numerics, *Mathematica* also provides an arbitrary-precision numerical engine: if accuracy is

important, *Mathematica* excels. As McCullough (2000, p.296) notes, “By virtue of its variable precision arithmetic and symbolic power, *Mathematica*’s performance on these reliability tests far exceeds any finite-precision statistical package”.

(iv) *Cross-platform and large user base*

Mathematica runs on a wide variety of platforms, including Mac, OS X, Windows, Linux, SPARC, Solaris, SGI, IBM RISC, DEC Alpha and HP-UX. This is especially valuable in academia, where co-authorship is common.

What is mathStatica?

mathStatica is a computer software package—an add-on to *Mathematica*—that provides a sophisticated toolset specially designed for doing mathematical statistics. It automatically solves the types of problems that researchers and students encounter, over and over again, in mathematical statistics. The **mathStatica** software is bundled free with this book (Basic version). It is intended for use by researchers and lecturers, as well as postgraduate and undergraduate students of mathematical statistics, in any discipline in which the theory of statistics plays a part.

Assumed Knowledge

How much statistics knowledge is assumed? How much *Mathematica* knowledge?

Statistics: We assume the reader has taken one year of statistics. The level of the text is generally similar to Hogg and Craig (1995). The focus, of course, is different, with less emphasis on theorems and proofs, and more emphasis on problem solving.

Mathematica: No experience is required. We do assume the reader has *Mathematica* installed (this book comes bundled with a fully-functional trial copy of *Mathematica*) and that the user knows how to evaluate $2+2$, but that’s about it. Of course, there are important *Mathematica* conventions, such as a knowledge of bracket types $()$, $[]$, $\{\}$, which are briefly discussed in Chapter 1. For the new user, the best approach is to try a few examples and the rest usually follows by osmosis ☺.

As a Course Textbook

This book can be used as a course text in mathematical statistics or as an accompaniment to a more traditional text. We have tried to pitch the material at the level of Hogg and Craig (1995). Having said that, when one is armed with **mathStatica**, the whole notion of what is difficult changes, and so we can often extend material to the level of, say, Stuart and Ord (1991, 1994) without any increase in ‘difficulty’. We assume that the reader has taken preliminary courses in calculus, statistics and probability. Our emphasis is on problem solving, with less attention paid to the presentation of theorems and their associated proofs, since the latter are well-covered in more traditional texts. We make no assumption about the reader’s knowledge of *Mathematica*, other than that it is installed on their computer.

In the lecture theatre, lecturers can use **mathStatica** to remove a lot of the annoying technical calculation often associated with mathematical statistics. For example, instead of spending time and energy laboriously deriving, step by step, a nasty expectation using integration by parts, the lecturer can use **mathStatica** to calculate the same expectation in a few seconds, in front of the class. This frees valuable lecture time to either explore the topic in more detail, or to tackle other topics. For students, this book serves three roles: first, as a text in mathematical statistics; second, as an interactive medium to explore; third, as a tool for tackling problems set by their professors—the book comes complete with 101 exercises (a solution set for instructors is available at www.mathstatica.com).

mathStatica has the potential to enliven the educational experience. At the same time, it is not a panacea for all problems. Nor should it be used as a substitute for thinking. Rather, it is a substitute for mechanical and dreary calculation, hopefully freeing the reader to solve higher-order problems. Armed with this new and powerful toolset, we hope that others go on to solve ever more challenging problems with consummate ease.

Acknowledgements

Work on **mathStatica** began in 1995 for an invited chapter published in Varian (1996). As such, our first thanks go to Hal Varian for providing us with the impetus to start this journey, which has taken almost five years to complete. Thanks go to Allan Wylde for his encouragement at the beginning of this project. Combining a book (print and electronic) with software creates many unforeseen possibilities and complexities. Fortunately, our publisher, John Kimmel, Springer's Executive Editor for Statistics, has guided the project with a professional savoir faire and friendly warmth, both of which are most appreciated.

Both the book and the software have gone through a lengthy and extensive beta testing programme. B.D. McCullough, in particular, subjected **mathStatica** to the most rigorous testing, over a period of almost two years. Marc Nerlove tested **mathStatica** out on his undergraduate classes at the University of Maryland. Special thanks are also due to flip phillips and Ron Mittelhammer, and to Luci Ellis, Maxine Nelson and Robert Kieschnick. Paul Abbott and Rolf Mertig have been wonderful sounding boards.

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